

The Cosmological Constant is a Chemical Potential*

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The statistical mechanics described by partition functions for quantum gravity that assume a finite number of degrees of freedom for a fixed finite spacetime region are discussed. It is shown that a cosmological constant arises naturally as a Lagrange multiplier needed to fix the 4-volume in the canonical ensemble and that in the grand canonical ensemble, the cosmological constant naturally emerges in terms of a volume normalized chemical potential as $\Lambda = -8\pi\mu$. This result is demonstrated classically by defining the free energy in terms of the classical action and calculating the change in free energy when a unit volume is added to a spacetime region having a fixed boundary.

I. INTRODUCTION

The cosmological constant is often interpreted to be a result of vacuum energy[1–3] and, as is well known, this leads to estimates of the cosmological constant many orders of magnitude larger than observations allow. The problem with interpreting the cosmological constant as vacuum energy has led to the introduction of alternative models[4–6] such as quintessence[7]. We will argue that the notion that the cosmological constant is determined by vacuum energy rests on an incorrect assumption regarding how Einstein’s equation arises as a classical approximation to an underlying quantum theory. In particular, Einstein’s equation follows as a stationary phase approximation to a quantum mechanical partition function. If there are an unlimited number of degrees of freedom in any region of spacetime, variation of the metric in the action is unrestricted and the resulting Einstein equation contains a stress-energy tensor that includes an absolute energy density. The zero-point energy would then appear to contribute to the stress-energy tensor. However, if we assume that there is only a finite number of degrees of freedom in any finite region of spacetime, the variation of the metric must be restricted. We will show that implementing such a restriction introduces an energy density scale so that the energy density appearing in the resulting Einstein equation can be regarded as a relative energy density that does not include vacuum energy.

In [8], we derived a partition function for quantum gravity where any finite spacetime region contains only a finite number of degrees of freedom. Here we study the consequences of the general structure of this theory for the cosmological constant. In Section II, we show that the cosmological constant naturally arises from the canonical ensemble as a Lagrange multiplier, λ , needed to enforce a fixed 4-volume constraint. That the cosmological constant could be a Lagrange multiplier was considered in [6] where it was argued that the approach is problematic because of the need to restrict the range of the Lorentzian time integral. Such difficulties do not arise in our approach, however, where we work in a local Euclidean spacetime region[8]. In Section III we discuss the grand canonical ensemble and relate the cosmological constant to a chemical potential. We then show in Section IV that the conclusion from the previous sections that $\lambda = -\mu = \Lambda/(8\pi)$ follows at the classical level by calculating the change in free energy when the 4-volume in a region with a fixed boundary is varied. We discuss some of the implications of our results in Section V where we argue that zero-point energy plays no role in determining the cosmological constant.

* This paper is available at <http://uvfinite.weebly.com/>

II. PARTITION FUNCTION FOR QUANTUM GRAVITY

In [8], we derived an expression for the canonical ensemble partition function that can be written in the form

$$Q(V, \Sigma) = \text{tr } e^{-S} \quad (1)$$

where V is the fixed 4-volume in a region M bounded by the 3-surface $\Sigma = \partial M$ and the trace involves integration over a finite number of metric, ghost, and matter degrees of freedom. The 4-volume was required to be held fixed, because it was assumed that there is a finite number of degrees of freedom for any given finite 4-volume and fixing the 4-volume allowed the partition function to be expressed in terms of a finite number of integrations. Classical general relativity results from taking the stationary phase approximation to the partition function with a Lagrange multiplier λ introduced to keep the volume fixed. The Euclidean spacetime action can be written

$$S = S_G + S_M \quad (2)$$

where S_G is the gravitational action, which may be taken to a suitably Wick-rotated version of the Einstein-Hilbert action[8, 9] and S_M is the action for matter fields. For considering the stationary phase approximation, we may take (in $d = 4$)

$$S_G = -\frac{1}{16\pi} \int_M d^4x \sqrt{g} R - \frac{1}{8\pi} \int_{\partial M} d^3x \sqrt{h} K \equiv S_B + S_S \quad (3)$$

where R is the Euclidean Ricci scalar, h is the determinant of the metric induced on ∂M , K is the extrinsic curvature of ∂M , and S_B and S_S refer to bulk and surface terms respectively. We may write the matter contribution as

$$S_M = - \int d^4x \sqrt{g} \mathcal{L}_m \quad (4)$$

Defining

$$T_{ab} = -\frac{2}{\sqrt{g}} \frac{\delta(\sqrt{g} \mathcal{L}_m)}{\delta g^{ab}} \quad (5)$$

and imposing the stationary phase condition

$$\frac{\delta}{\delta g_{ab}} (S_G + S_M + \lambda V) = 0 \quad (6)$$

results in the Einstein's equation

$$G^{ab} = 8\pi(T^{ab} - \lambda g^{ab}) \quad (7)$$

so that $8\pi\lambda$ is the cosmological constant. We will show in the next section that the cosmological constant is related to a suitably normalized chemical potential.

III. GRAND CANONICAL PARTITION FUNCTION

Here we consider a grand canonical ensemble partition function. Before describing the thermodynamics, we first discuss the geometry of the statistical mechanical system. As we discussed in [8], a Lorentzian metric tensor can always locally be written as

$$d^2s = -dt^2(1 + n(\vec{x})) + (\delta_{ij} + g_{ij}(\vec{x}))dx^i dx^j + v_i(\vec{x})dx^i dt. \quad (8)$$

where $n(\vec{x})$, $g_{ij}(\vec{x})$ and $v_i(\vec{x})$ are quadratic in spatial coordinates and are independent of the time coordinate t and cubic and higher terms have been dropped. This can be understood as the Wick rotation of the Euclidean metric

$$d^2s = d\tau^2(1 + n(\vec{x})) + (\delta_{ij} + g_{ij}(\vec{x}))dx^i dx^j + v_i(\vec{x})dx^i d\tau. \quad (9)$$

where Wick rotation takes $t \rightarrow -i\tau$ and $v_i \rightarrow iv_i$. The τ coordinate is naturally interpreted as the Euclidean time parameter in a thermodynamic description of spacetime degrees of freedom with τ running from 0 to the inverse temperature β . It is convenient to define Σ as follows. Define a 2-surface Σ_0 at a time slice $\tau = 0$ and extend this surface along τ to form a cylinder. Σ may be defined as the 3-surface bounding the 4-volume contained within the extended surface Σ_0 and the surfaces $\tau = 0$ and $\tau = \beta$.

To define the grand canonical ensemble partition function, we start with the canonical ensemble partition function and sum over discrete possible 4-volumes, V , with the partition function weighted by a factor of $e^{\mu V}$ where μ is the chemical potential with a volume normalization. The grand canonical partition function can then be written as

$$Z(\mu, \beta, \Sigma_0) = \text{tr } e^{-S_E + \mu V} \quad (10)$$

where the trace now include a sum over 4-volume and the number of terms included in the expansion of the metric for a given 4-volume depends on the 4-volume. The stationary phase approximation then gives Einstein's equation with $\Lambda = -8\pi\mu$.

We will now show that the chemical potential μ appearing in this partition function can be related to a partial derivative of the free energy by taking the differential of $\ln Z(\mu, \beta, \Sigma_0)$ and comparing to the first law of thermodynamics. Even though this analysis is very similar to the traditional case[10], we will work through this in some detail since our normalizations differ from that of standard statistical mechanics.

We can define a thermodynamic average of the energy by

$$\bar{E} = -\frac{\partial}{\partial \beta} \ln Z(\mu, \beta, \Sigma_0) \quad (11)$$

and similarly it follows from equation (10) that the average 4-volume is

$$\bar{V} = \frac{\partial}{\partial \mu} \ln \tilde{Z}(\mu, \beta, \Sigma_0). \quad (12)$$

The pressure in conventional statistical mechanics would be defined as $P = \frac{1}{\beta} \frac{\partial}{\partial V^{(3)}} \ln \tilde{Z}$. Here, the boundary Σ_0 plays the role of volume in ordinary statistical mechanics. We can define a pressure as

$$P = \frac{1}{\beta} \frac{\partial}{\partial V_{\Sigma_0}} \ln Z(\mu, \beta, \Sigma_0) \quad (13)$$

where the volume derivative $\frac{\partial}{\partial V_{\Sigma_0}}$ is defined by

$$\frac{\partial}{\partial V_{\Sigma_0}} f(\Sigma_0) = \frac{f(\Sigma_1) - f(\Sigma_0)}{V_{\Sigma_1} - V_{\Sigma_0}} \quad (14)$$

where Σ_1 is the surface Σ_0 with each point moved a uniform infinitesimal distance normal to Σ_0 such that the 3-volume between Σ_1 and Σ_0 is $dV = V_{\Sigma_1} - V_{\Sigma_0}$. The differential of $\ln Z(\mu, \beta, \Sigma_0)$ is then

$$d \ln Z = -\bar{E} d\beta + \beta P dV_{\Sigma_0} + \bar{V} d\mu \quad (15)$$

from which we obtain

$$d\bar{E} = \frac{1}{\beta} d(\ln Z + \beta\bar{E} - \mu\bar{V}) - PdV_{\Sigma_0} + \frac{1}{\beta} \mu d\bar{V}. \quad (16)$$

Writing $T = 1/\beta$ and $S = \ln Z + \beta\bar{E} - \mu\bar{V}$, we have

$$d\bar{E} = TdS - PdV_{\Sigma_0} + T\mu d\bar{V}. \quad (17)$$

This can be recognized as the first law of thermodynamics with a modification to the normalization of the chemical potential term. It follows from equation (17) that the chemical potential can be written in terms of the Helmholtz free energy, $A = \bar{E} - TS$, as

$$\mu = \beta \left(\frac{\partial A}{\partial V} \right)_{\beta, \Sigma_0} \quad (18)$$

where we have dropped the bar on V for simplicity of notation and it should be understood that the thermodynamically averaged V is used here. We see that μ is β times the change in free energy when a unit of 4-volume is added to the volume bounded by the surface Σ holding the boundary and temperature fixed. If we define a 3-volume as $V^{(3)} \equiv V/\beta$ we can write

$$\mu = \left(\frac{\partial A}{\partial V^{(3)}} \right)_{\beta, \Sigma_0} \quad (19)$$

and we see that μ can be described as the change in free energy when a unit of 3-volume is added to the volume bounded by the 2-dimensional surface Σ_0 holding the boundary and temperature fixed. This is naturally interpreted as a volume-normalized chemical potential. Expressing the free energy in terms of the canonical ensemble partition function, we find

$$\mu = -\frac{\partial}{\partial V} \ln Q(V, \Sigma). \quad (20)$$

Since we have found $\Lambda = -8\pi\mu$ and $\Lambda = 8\pi\lambda$, it follows that

$$\lambda = \frac{\partial}{\partial V} \ln Q(V, \Sigma) = -\mu. \quad (21)$$

In the next section, we will calculate the partial derivative of the canonical ensemble partition function at the classical level and show that equation (21) holds.

IV. CHEMICAL POTENTIAL IN THE CANONICAL ENSEMBLE

Consider a slowly varying, spherically symmetric distribution of matter that can be described as an ideal fluid with density ρ and pressure p . We will calculate the free energy in a finite volume region where the length scale of the volume is large compared to L_d but small compared to the length scale on which ρ and p vary. We will hold the metric, ρ and p fixed on the boundary of our volume and vary the volume inside the boundary by varying the metric. We do this at the stationary phase level of approximation for the free energy and calculate the chemical potential using equation (20).

In keeping with the spirit of the local thermodynamic equilibrium approximation, we assume that the length scale, L_V associated with our fixed volume region is small compared to the curvature length scale L_c . If we were to calculate $\frac{\partial}{\partial V} \ln Q(V, \Sigma)$ as a power series in ρ , p and λ only the linear terms would contribute in the limit that $L_V \ll L_c$ since higher order terms contain factors of

L_V/L_c . The derivative with respect to V , however, can remove a factor of λ from $\frac{\partial}{\partial V} \ln Q(V, \Sigma)$ so to guarantee that we get all linear terms in $\frac{\partial}{\partial V} \ln Q(V, \Sigma)$, we will keep terms to quadratic order in $\ln Q(V, \Sigma)$.

We cannot assume that ρ and p are constant since we would not be able to satisfy Einsteins equation if this were the case. In particular, the Tolman-Oppenheimer-Volkoff equation[11], which follows from the Einstein equation, requires that the pressure depend quadratically on radial coordinate \tilde{r} for small \tilde{r} for a slowly varying spherically symmetric distribution of matter. If we assume that the matter obeys an equation of state such that the pressure is a function of energy density then the energy density must also vary quadratically with \tilde{r} . We may therefore write

$$\rho = \rho_0 - \rho_1 \tilde{r}^2 = \rho_b - \rho_1(\tilde{r}^2 - \tilde{r}_b^2) \quad (22)$$

where \tilde{r}_b is the value of \tilde{r} at the boundary, ρ_0, ρ_b are the densities at $\tilde{r} = 0$ and at the boundary respectively, and ρ_1 is a parameter describing the variation of ρ . Similarly, we may write

$$p = p_0 - p_1 \tilde{r}^2 = p_b - p_1(\tilde{r}^2 - \tilde{r}_b^2). \quad (23)$$

From the Tolman-Oppenheimer-Volkoff equation, we have to quadratic order

$$p_1 = 2\pi(p_b + \rho_b)(p_b + \frac{1}{3}\rho_b - \frac{2}{3}\lambda) \quad (24)$$

The density variation parameter can be related to p_1 by the equation of state

$$\rho_1 = \frac{p_1}{dp/d\rho} \quad (25)$$

where $dp/d\rho$ can be taken to be a constant in evaluating ρ_1 to quadratic order.

The solution to Einstein's equation can be written as[11]

$$ds^2 = e^{2\phi} d\tau^2 + \left(1 - \frac{2m(\tilde{r})}{\tilde{r}}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2 d\Omega^2 \quad (26)$$

where

$$\frac{d\phi}{d\tilde{r}} = \frac{m(\tilde{r}) + 4\pi\tilde{r}^3(p - \lambda)}{\tilde{r}(\tilde{r} - 2m(\tilde{r}))}, \quad (27)$$

$$m(\tilde{r}) = 4\pi \int_0^{\tilde{r}} (\rho(r') + \lambda) r'^2 dr' \quad (28)$$

and $\tilde{r} = (A/4\pi)^{1/2}$ where A is the area of the 2-sphere of radius \tilde{r}

We consider an unperturbed geometry having some 4-volume $V(\lambda)$ and consider a perturbed geometry with a 4-volume $V(\lambda + \delta\lambda)$. For both geometries, the metrics can be written in the form of equation (26). A radial coordinate $\tilde{r} = (A/4\pi)^{1/2}$ can be defined for each geometry, however we cannot simply identify coordinates in the two geometries except at the boundary where the geometry is held fixed. For the angular components, matching the metric on the boundary is achieved simply by the condition that $\tilde{r} = r_b$ at the boundary in both cases. For the time component, this is achieved by imposing a common boundary condition in equation (27). For the radial component, we cannot set $g_{\tilde{r},\tilde{r}}$ in the unperturbed geometry equal to $g_{\tilde{r},\tilde{r}}$ in the perturbed geometry on the boundary. This is because $g_{\tilde{r},\tilde{r}}$ measures distances off of the boundary and we cannot, in general, identify points off of the boundary labeled by the coordinate \tilde{r} in the two different geometries. However, the choice of coordinates is arbitrary and we need not express

the metric in both geometries using the $\tilde{r} = (A/4\pi)^{1/2}$ coordinate. Let write the metric in the unperturbed geometry using the radial coordinate as $r \equiv \tilde{r}$ and in the unperturbed geometry we write the metric using a coordinate r' defined by

$$\tilde{r} = \tilde{r}(r') = r' + \gamma(r'^2 - r_b^2)(r'/r_b)^2. \quad (29)$$

Note that on the boundary $\tilde{r} = (A/4\pi)^{1/2} = r'$ which can be identified with $r = (A/4\pi)^{1/2}$ on the boundary since the metric and hence the area A is held fixed on the boundary. We can choose γ so that the metric expressed in terms of r' is equal to the metric expressed in terms of r on the boundary, i.e., $g_{rr} = g_{r'r'}$ on the boundary. We can then identify r' with r up to an arbitrary diffeomorphism that preserves the boundary and the metric on the boundary. Since the theory is invariant under such boundary preserving diffeomorphisms, we will henceforth drop the distinction between r' and r .

Using equations (22) and (23) in equations (26) - (28), the metric to second order can be written as

$$ds^2 = c(1 + a\tilde{r}^2 + \frac{1}{2}(a^2 + b)\tilde{r}^4)d\tau^2 + (d\tilde{r}/dr)^2(1 + A\tilde{r}^2 + B\tilde{r}^4)dr^2 + \tilde{r}^2 d\Omega^2 \quad (30)$$

where $\tilde{r} = r$ for the unperturbed geometry, \tilde{r} is given by equation (29) with r' replaced with r for the perturbed geometry,

$$a = \frac{4\pi}{3}(\rho_b + \lambda + \rho_1 r_b^2) + 4\pi(p_b - \lambda + p_1 r_b^2) \quad (31)$$

$$b = -\frac{4\pi}{5}\rho_1 - 4\pi p_1 + 2\left(\frac{4\pi}{3}\right)^2(\rho_b + \lambda)(\rho_b + 3p_b - 2\lambda), \quad (32)$$

$$A = \frac{8\pi}{3}(\rho_b + \lambda + \rho_1 r_b^2), \quad (33)$$

$$B = -\frac{8\pi}{5}\rho_1 + \left(\frac{8\pi}{3}\right)^2(\rho_b + \lambda)^2, \quad (34)$$

and c is a constant that is determined by the boundary conditions. We will denote values for the unperturbed geometry with a 0 subscript. For the unperturbed geometry, we take $c_0 = 1$. The value of c in the perturbed geometry is then given by requiring the metric to remain fixed on the boundary. $c^{1/2}$, which will be need shortly, can be expresses to second order as

$$c^{1/2} = 1 + \frac{1}{2}(a_0 - a)r_b^2 + \frac{1}{8}(a_0 - a)^2 r_b^4 + \frac{1}{4}(b_0 - b)r_b^4 \quad (35)$$

Finally, γ is determined by matching g_{rr} at the boundary. We find

$$\gamma = \frac{1}{2r_b} \left(\left(\frac{1 + A_0 r_b^2 + B_0 r_b^2}{1 + A r_b^2 + B r_b^2} \right)^{1/2} - 1 \right) \quad (36)$$

Since in the stationary phase approximation (in our sign convention) $Q = e^{-S}$,

$$\mu = -\frac{\partial}{\partial V} \ln Q(V, \Sigma) = \frac{\partial}{\partial V} S \quad (37)$$

and we can calculate the chemical potential by taking the volume derivative of the total action evaluated at the stationary phase condition. We calculate three contributions to the chemical potential. The first is the contribution from the volume term in the action. This can be calculated using the trace of Einsteins equation

$$R = 32\pi\lambda + 8\pi(\rho - 3p). \quad (38)$$

The contribution to the free energy from this term can be obtained by substituting the previous expressions for ρ and p and using the volume element determined from the above metric. The derivatives with respect to volume can be written as

$$\frac{\partial}{\partial V} = \frac{1}{\partial V / \partial \lambda} \frac{\partial}{\partial \lambda}. \quad (39)$$

After some algebra, we find that the contribution to the chemical potential from the bulk term is

$$\mu^{S_B} = -\frac{3}{2\pi r_b^2} + 2\lambda - \frac{33}{70}p_b + \frac{107}{70}\rho_b - \frac{9}{35} \frac{(\rho_b + p_b)}{dp/d\rho}. \quad (40)$$

There is also a contribution to the free energy from the term from the surface part of the action. This can be calculated from

$$\int_{\partial M} d^3x \sqrt{h} K = \frac{1}{g_{rr}^{1/2}} \frac{\partial}{\partial r} A_r|_{r=r_b} \quad (41)$$

where A_r is determined from the metric. Using equation (30) in equation (41) and applying equation(39), we obtain (again, after some algebra)

$$\mu^{S_S} = \frac{3}{2\pi r_b^2} - 3\lambda + \frac{15}{14}p_b - \frac{27}{14}\rho_b + \frac{9}{35} \frac{(\rho_b + p_b)}{dp/d\rho}. \quad (42)$$

Finally we have a contribution from the matter part of the action. This can be derived from equation (5). We have

$$\frac{\partial}{\partial \lambda} S_M = -\frac{\partial}{\partial \lambda} \int d^4x \sqrt{g} \mathcal{L}_m = \frac{1}{2} \int d^4x \sqrt{g} T^{\mu\nu} \frac{\partial}{\partial \lambda} g_{\mu\nu}. \quad (43)$$

Using equation (39) and using equation(30) in equation(43) with the stress-energy tensor of an ideal fluid, we find that the contribution of the matter part of the action to the chemical potential of the spacetime degrees of freedom can be written as

$$\mu^{S_M} = -\frac{3}{5}p_b + \frac{2}{5}\rho_b. \quad (44)$$

Adding these three terms together we have simply

$$\mu = \mu^{S_B} + \mu^{S_S} + \mu^{S_M} = -\lambda = -\frac{\Lambda}{8\pi}. \quad (45)$$

V. COSMOLOGICAL CONSTANT

A common interpretation of the cosmological constant is that it describes the energy density of the vacuum. From the point of view of the present work, this is a misconception that arises from the incorrect notion that general relativity depends on an absolute energy density rather

than a relative energy density. As noted earlier, this is consistent if there are an unlimited number of degrees of freedom in any finite region of spacetime. In our approach, however, this is not the case and a relative energy enters into general relativity just like every other area of physics. In particular, it is the energy density relative to a chemical potential that is relevant in general relativity.

To explore this further, consider an analogy with condensed matter physics. For non-interacting bosons, the occupation number can be written as

$$n_k = \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1}, \quad (46)$$

where ϵ_k are the energy levels of the system. If we don't include zero-point energy in the definition of ϵ_k then we should not include it in the definition of μ . The zero-point energy represents an arbitrary zero of energy and as long as ϵ_k and μ are defined relative to the same choice for the zero of energy, the occupation number does not depend on this choice. The same is true in general relativity. In synchronous coordinates in Lorentzian spacetime ($g_{00} = -1$), we have

$$G_{00} = 8\pi T_{00} + \Lambda = 8\pi(T_{00} - \mu). \quad (47)$$

We see that as long as T_{00} does not include zero-point energy then neither should μ . If one insists on adding a constant energy density to T_{00} to represent zero-point energy, one should add the same constant to μ so that T_{00} and μ are defined relative to the same arbitrary zero of energy. But then this constant energy density does not enter into Einstein's equation. Since the conventional definition of T_{00} in general relativity does not include zero-point energy, then neither should μ . More generally, it is only the difference between T_{ab} and $-g_{ab} \mu$ which enters Einstein's equation and we should either include zero-point contributions to both T_{ab} and $-g_{ab} \mu$ or to neither. Problems only occur if we attempt to include zero-point contributions in one of these terms and not the other.

A general feature of discrete spacetime theories[8] is the existence of a finite number of degrees of freedom associated with any given finite region of spacetime and as we have seen a cosmological constant that is unrelated to zero point energy enters into such theories. The observed evidence for a cosmological constant with a magnitude many orders of magnitude below the Planck scale can be taken as an indication that discrete degrees of freedom underlie spacetime. Note that although our approach divorces zero-point energy from the cosmological constant and thus explains why naive estimates are off by many orders of magnitude, it does not explain why the cosmological constant takes on the particular value that it does and it does not address the coincidence problem, though it is not clear how serious a problem this is[12].

VI. CONCLUSION

We have shown that if one assumes that there should be a finite number of degrees of freedom in any finite spacetime region, then the existence of a cosmological constant follows quite generally from the partition function written as a sum over the degrees of freedom in a fixed 4-volume where a Lagrange multiplier must be introduced to fix the 4-volume when the metric is varied. The cosmological constant written in terms of the Lagrange multiplier is simply $\Lambda = 8\pi\lambda$.

If we assume that a thermodynamic interpretation of the finite number of degrees of freedom is appropriate, as we argued in [8], then the cosmological constant is naturally interpreted in terms of a volume-normalized chemical potential. We showed that $\Lambda = -8\pi\mu$. This relation follows directly from the grand canonical partition function. We explicitly verified that $\mu = -\lambda$ for a classical, slowly-varying, spherically symmetric distribution of matter by calculating $\ln Q$ in the stationary phase approximation and differentiating with respect to 4-volume to obtain the chemical potential.

We discussed the implication of the interpretation of the cosmological constant as a chemical potential. In particular, we argued that this implies that there is no zero-point contribution to the cosmological constant, contrary to a popular misconception. The theory presented here does not shed any light on the particular value that the cosmological constant takes other than to show that estimates that are off by 60 or 120 orders of magnitude are based on an incorrect understanding of how zero-point energy enters, or fails to enter, Einstein's equation. From the point of view presented here, the cosmological constant can be interpreted as a parameter that describes how much 4-volume in empty spacetime is contained inside a region bounded by some fixed boundary (i.e., how flat empty spacetime is), just as the chemical potential for particles in ordinary statistical mechanics is a parameter that describes how many particles are within a fixed region.

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